Rayleigh–Taylor growth at decelerating interfaces

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The number of linear e-foldings of Rayleigh–Taylor instability growth is calculated for several cases of interest to experiment design. The planar, Sedov–Taylor case produces maximum Rayleigh–Taylor growth. [DOI: 10.1063/1.1418434]

The Rayleigh–Taylor (RT) instability[1,2] is important in a number of research contexts. For the design or analysis of experiments and observations, it is often useful to know the growth exponent of an interface, even when the growth is strongly nonlinear. The growth exponent is defined here as the number of linear e-foldings by which initial perturbations at the interface would be amplified, if they remained in the regime of linear amplification. Even though the interfaces in actual experiments often develop very nonlinear structures, the growth exponent can still be useful, as a characterization of the degree of nonlinearity. Here we are concerned with the calculation of growth exponents, under simple assumptions, at a decelerating interface. Decelerating interfaces are important in any impulsively driven system, including supernovae,[3] stellar eruptions, and laboratory studies of hydrodynamic turbulence.[4]

Here we will represent the growth exponent, G, as the integral of the growth rate, γ, over time,

$$G=\int_{t_0}^{t}\gamma(t')dt'. \tag{1}$$

The growth rate for the classical RT instability at the interface is $\gamma=\sqrt{Akg}$, in which A is the Atwood number, g is the acceleration, and k is the wave number of the perturbation on the surface. The growth factor is often related to the distance that an interface has moved. In the common circumstance of constant acceleration from rest, or deceleration to rest, the result is very simple. When the interface accelerates to rest (or from rest), the distance traveled, D, is $\frac{1}{2}gt^2$, from which one finds the usual result for the growth exponent, $G=\sqrt{2AkD}$. Thus, the growth exponent is determined by D/λ, where λ is the wavelength of the perturbation.

Decelerating interfaces are important, but often do not experience constant deceleration to rest. In both laboratory experiments and exploding stars, for example, the interface may be accelerated by a shock wave and then may decelerate as the shocked, moving plasma accumulates more and more mass. However, the interface may not decelerate to a stop on the time scale of interest for the experiment or for the natural system. Here we develop simple formulas that provide estimates for some useful cases involving a decelerating interface. For simplicity, we ignore any contribution from initial growth due to the Richtmyer–Meshkov instability.[5,6]

First, we consider the case of an interface that experiences a constant deceleration, but not to rest. The interface is assumed to start at velocity v_0 and to decelerate for a time t, so that $D=v_0t-\frac{1}{2}gt^2$. The system would stop in a time $t=v_0/g$, after traveling a distance $D=D_{\text{stop}}=\frac{1}{2}v_0^2/g$. If one designates the actual distance traveled by the interface as a fraction, η, of the stopping distance, so $D=\eta D_{\text{stop}}$, then one finds

$$G=\sqrt{2AkD}\times(1-\sqrt{1-\eta})/\sqrt{\eta}. \tag{2}$$

This is the key result for this case. For a given distance D that the interface travels, the growth exponent is a fraction of the fully stopped growth exponent, given by $(1-\sqrt{1-\eta})/\sqrt{\eta}$. The implication is that such an experiment, which typically would be limited by two-dimensional effects to some maximum value of D, should nearly stop the interface in that distance in order to achieve the largest possible growth exponent.

A second common case is that of Sedov–Taylor deceleration,[7,8] in which the accumulation of mass causes the deceleration. In the typical case, mass accumulation causes a shock wave to decelerate as a power of time, so that $D_S(t)=D_0(t/t_0)^{\beta}$, where D_S is the position of the shock, D_0 is the distance the shock has traveled at a reference time t_0, and where $\beta=2/5$, 1/2, and 2/3 for three-dimensional (3D), two-dimensional (2D), and one-dimensional (1D) expansions, respectively. An unstable interface through which such a shock wave has passed decelerates more quickly than the shock wave does, but not dramatically so until it has slowed substantially. Eventually it comes to rest and recedes. To obtain results that are suitable for experimental scoping, we will take $D(t)=D_S(t)$. Exactly how good this approximation is will have to be determined by simulations for specific cases. For one published experiment, it is excellent.[9] For a planar case that we have simulated, it applies for about half the distance the interface travels.

For each geometry, one can infer the acceleration from $D(t)$ and can evaluate Eq. (1) to obtain G. In the 2D and 3D cases, one must also allow for the decrease in k from its initial value, k_0, which satisfies $k_0/k=D/D_0$. In the 1D case, one finds $G=\sqrt{2AkD}$, just as in the case of constant deceleration to rest. (The decreased acceleration is compensated for by an increase of time as the interface slows.) In the 2D and 3D cases, one finds

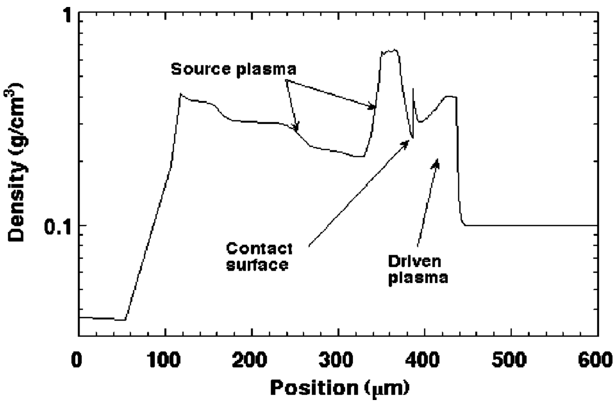

FIG. 1. The density profile from a system in which a dense layer of plasma drives a shock wave into low-density foam. The source plasma is produced by laser irradiation of an initial plastic layer, initially separated by a vacuum gap from the driven, low-density foam.

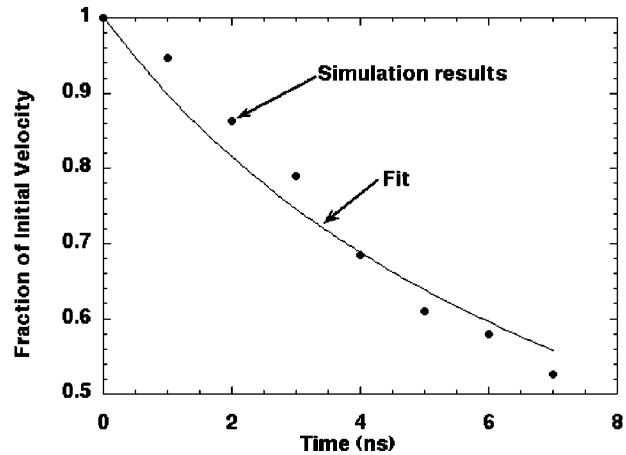

FIG. 2. The slowing of the interface between the source plasma and the driven plasma of Fig. 1 is shown. The curve shows the model described in the text.

$$G=\sqrt{2Ak_0D_0}\times 0.35\left(1-\left(\frac{D_0}{D}\right)^{1/\beta}\right)$$
$$=\sqrt{2Ak_0D}\times 0.35\sqrt{\frac{D_0}{D}}\left(1-\left(\frac{D_0}{D}\right)^{1/\beta}\right). \quad (3)$$

In these cases the decrease in wave number has a very large effect, assuring that the growth remains much smaller than it does in the 1D, planar case. It is also worth noting that the maximum growth exponent, for large expansions, can be written $G=0.35\sqrt{2Al}$, where l is the mode number of the perturbation.

A third case of interest, we consider the deceleration of a layer of dense material that has impacted a second, lower-density material. This type of situation may result when a flyer plate has impacted a very thick sample, when a flowing plasma has been used to launch a shock into a sample,[10] or after a blast wave is used to accelerate an interface.[11] In this case, the deceleration decreases as the moving system accumulates more mass. Here we offer a very simple model of such a system, show that it produces the approximate motion observed in a simulation, and discuss the implications for the growth exponent.

We assume that the matter that has provided the energy to the system, but is now being decelerated, has an areal mass density $\rho_1 d_0$, where ρ_1 is an effective density and d_0 is an effective thickness. In simple cases, such as the behavior of a massive flyer plate striking a very-low-density medium, ρ_1 and d_0 may correspond to an actual density and thickness. In other cases, such as when a shock wave is driven through low-density matter by an incoming, stagnating plasma flow, as in a supernova remnant,[12] the situation is less well defined. Here a clump of dense material accumulates and is decelerated, and we see below that the time behavior can be approximated by assuming the driving mass to have some effective areal mass density, $\rho_1 d_0$. This driving mass is decelerated in reaction to the rate of increase of momentum in the low-density matter, of initial density ρ_2. We can designate the postshock fluid velocity, in the low-density matter, by v. We can also approximate the medium as a polytropic gas with index γ, so that the shock velocity is $\{(\gamma+1)/2\}v$. One then finds the deceleration, g, from

$$(\rho_1 d_0)g=\frac{\gamma+1}{2}\rho_2 v^2. \quad (4)$$

For deceleration from some initial velocity, v_0, beginning at time $t=0$, to velocity v, at later time t, one can integrate g to find

$$v=\frac{v_0}{1+\frac{(\gamma+1)}{2}\frac{\rho_2}{\rho_1}\frac{v_0 t}{d_0}}=\frac{v_0}{1+\alpha t}, \quad (5)$$

in which we define the parameter, α, as indicated. Thus, one expects the velocity in such a system to decrease approxi-

TABLE I. Summary of growth exponents.

Case	Growth exponent, G	Definitions
Constant deceleration to rest	$\sqrt{2AkD}$	
Constant deceleration for distance D	$\sqrt{2AkD}\times(1-\sqrt{1-\eta})/\sqrt{\eta}$	$\eta=2Dg/v_0^2$
Planar Sedov–Taylor	$\sqrt{2AkD}$	
Diverging Sedov–Taylor	$\sqrt{2Ak_0D}\times 0.35\sqrt{\frac{D_0}{D}}\left(1-\left(\frac{D_0}{D}\right)^{1/\beta}\right)$	$\beta=2/5$ for 3D, $\beta=1/2$ for 2D
Massive flyer plate	$\sqrt{2AkD}\times\sqrt{D/(2v_0/\alpha)}$	$v=v_0/(1+\alpha t)$

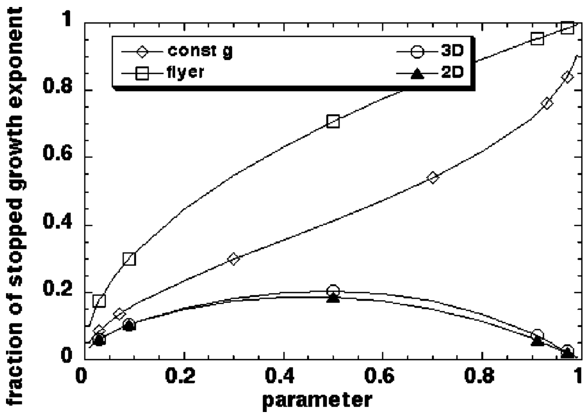

FIG. 3. For four cases, this figure shows the growth exponent as a fraction of the standard result, $\gamma t = \sqrt{2AkD}$. The abscissa is (a) the fraction of the stopping distance, η, for constant deceleration, (b) $\sqrt{D/(2v_0/\alpha)}$ for the flyer plate, and (c) the expansion ratio, D_0/D, for the two diverging cases. Thus, increasing distance moves the result to the left for the diverging systems and to the right for the other two cases. For constant deceleration to rest and for the planar Sedov–Taylor case, the growth exponent is $\sqrt{2AkD}$ at all distances.

mately as $(1+\alpha t)^{-1}$. Figures 1 and 2 show one example, similar to an actual experiment,[10] for which this model remains a good approximation. In this case, ejecta from shocked matter form an incoming, stagnating plasma flow that accumulates to drive a shock into lower-density matter. Specifically, the HYADES code[13] simulates the irradiation of a 100 μm thick plastic layer ($1.0\,\mathrm{g/cm^3}$) by a laser with an irradiance of $5\times10^{14}\,\mathrm{W/cm^2}$. Behind the plastic is a 150 μm vacuum gap followed by C foam of density $0.1\,\mathrm{g/cm^3}$. Figure 1 shows the structure of the density profile, with a dense layer of plasma pushing upon and driving a shock wave through the less dense medium in front of it. Figure 2 shows the decrease in the interface velocity with time. For this example, $\alpha = 1.1\times10^8\,\mathrm{s^{-1}}$ and $v_0 = 4.8\times10^6$ cm/s. The model of Eq. (5) is seen to be a reasonable approximation of the deceleration. The product αt reaches a maximum of about 1 in the 10 ns duration of the experiment. For comparison, one can calculate the time when the accelerated mass equals the driving mass, $\rho_1 d_0$, which will mark the transition to Sedov–Taylor behavior. This will occur at about 70 ns.

For a planar system described by Eq. (5), one can find the growth rate and integrate Eq. (1) to obtain

$$G = \sqrt{2AkD}\sqrt{\frac{D}{(2v_0/\alpha)}}. \tag{6}$$

This is typically much less than $\sqrt{2AkD}$. For the case just discussed, $G/\sqrt{2AkD}$ varies from 0.2 to 0.6.

In conclusion, we have provided formulas that can be used to estimate the growth exponent of an interface for some common cases. Table I summarizes these results, and Fig. 3 shows the fraction of $\sqrt{2AkD}$ that is reached in various cases. Two important conclusions are supported by this comparison. First, growth is very much reduced in diverging systems. Second, the experimental approach that will produce the greatest degree of nonlinear evolution is the planar Sedov–Taylor system. This has clear implications for experiment design.

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